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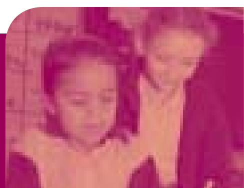
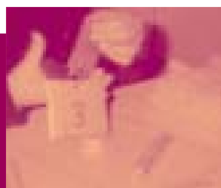
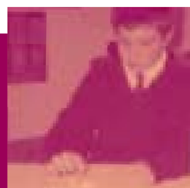
Understanding place value:
A case study of the Base Ten Game







A project funded under the Australian Government's Numeracy Research and Development Initiative
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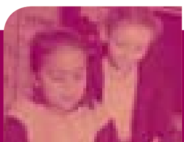
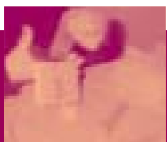


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Executive Summary

A major policy objective of the Australian Government is to ensure that all students attain sound foundations in literacy and numeracy. In 1997 all Education Ministers agreed to a National Literacy and Numeracy Plan that provides a coherent framework for achieving improvement in student literacy and numeracy outcomes. The 1999 Adelaide Declaration on National Goals for Schooling in the Twenty-First Century contains the national literacy and numeracy goal that *students should have attained the skills of numeracy and English literacy, such that every student should be numerate, able to read, write, spell and communicate at an appropriate level*.

In support of the numeracy component of the National Plan, the Australian Government implemented the Numeracy Research and Development Initiative in 2001. This Initiative consisted of two complementary strands—a national project strand and a strategic States and Territory projects strand. *Understanding place value: a case study of the Base Ten Game* project is one of ten strategic research projects undertaken by State and Territory education authorities across Australia. The purpose of these projects was to investigate a broad range of teaching and learning strategies that lead to improved numeracy outcomes.

Understanding place value: A case study of the Base Ten Game was the research focus of nine teachers from five diverse schools in the independent sector in South Australia. The project explored the role of a commonly used teaching activity, referred to in this report as the Base Ten Game, in developing children's understanding of our number system. This activity, based on a game described by Pengelly (1991) was selected because it models the structure of the number system, with students using a place value board and concrete materials to explore progressively larger, or smaller, quantities. The research project was conducted over Terms 2 and 3 in one school year.

Critical factors in developing students' understanding of the number system identified by the project were:

- The need for teachers to develop their own knowledge of the base ten number system to enable them to identify the learning needs of their students;
- Once teachers had developed their own relational understanding of the number system, they were better able to:
    - discover what each student already knew about base ten;
    - diagnose any misconceptions that a student may have developed;
    - offer learning activities that enabled students to build their knowledge about the number system;
    - adapt learning activities to meet the individual learning needs of the diversity of students in the class;
- Concrete materials, such as those used in the Base Ten Game, can make a significant contribution to the development of students' conceptual and procedural knowledge about the number system across all year levels;
- Any one set of concrete materials or any one teaching activity highlights only certain aspects of the number system. A range of materials and activities, chosen according to the features of the number system that they highlight, is required to develop a deeper understanding of the number system;
- The Base Ten Game is a valuable core activity for students of all year levels who are still trying to make sense of the structure of the number system. Its usefulness will be enhanced by the addition of complementary activities which both support the ideas being developed through the Base Ten Game, and which look at the same ideas in a different way (see Appendix 6).

Chapter 1: Introduction

Background to the study

The importance of understanding the number system

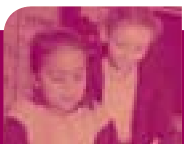
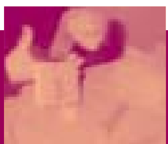
The ability to understand and to use the number system is crucial to the development of numeracy, and therefore fundamental to primary school mathematics (Shuard, 1982; Pengelly, 1991). Numeracy has been described as "the ability to understand, critically respond to and use mathematics in different social, cultural and work contexts" (The South Australian Curriculum, Standards and Accountability Framework, 2001, p. 225). The number aspect of numeracy plays a fundamental role in all aspects of people's lives as it is used to describe, represent, and compare aspects of our world. An understanding of the number system and how it works enables people to use the patterns, relationships and connections that exist within and between numbers to enhance their interactions with quantity, data, measurements, and chance events. This in turn enables them to participate fully in all aspects of life involving numbers, and works towards supporting them in life long learning to operate effectively and ethically in a rapidly changing society. A highly developed number sense can enhance people's lives on all levels, from managing their daily existence to engaging in critical examinations of social, environmental, political and cultural issues from a numerical perspective.

Difficulties with place value

Ideas about place value build on the broader ideas about the number system, and these ideas are complex and difficult for children to learn (Kamii, 1985; Ross, 1989). Many children are not aware, even at the most basic level, of the purpose or usefulness of our place value system, and need more active help in developing an adequate understanding of the structure of the number system (Thomas 1996).

Kamii (1986) cited results of research in the United States, Canada and Switzerland which found that most students in the first and second years of schooling do not understand that the 1 in 16 indicates that there is 1 ten, and that many students in Years 3 and 4 do not understand place value. Being able to put out the correct number of single units for a number such as 16, or writing the correct numeral 16 for sixteen objects does *not* constitute place value understanding. They might see the 16 as 16 separate entities and not as being made up of one ten and six ones. Even though many students in Reception or Year 1 can tell that 61 is greater than 16, they might do that because they know that 61 comes after 16 in the counting sequence – they don't see that 61 is greater because it is more than 6 tens, and 16 is less than two tens.

In a study by Thomas (1996), it was reported that most children master place value in numbers with up to four digits by Year 3, but that the progress after that is much slower. At Year 6 level, there were still 42% of students who could not name the ten thousands place. Many students in Years 2 to 6 were unable to use the structure of the number system to help them count in recursive groups of tens, and many showed poor visualisations of the array structure of the number system 1-100. Many students counted on from one number to another only in ones and didn't count on in tens and ones, even when presented with a 100 chart. Even in Years 5 and 6, there were considerable numbers of students who did not consistently use multiplicative relationships (see Figure 1) such as one hundred being ten times ten.

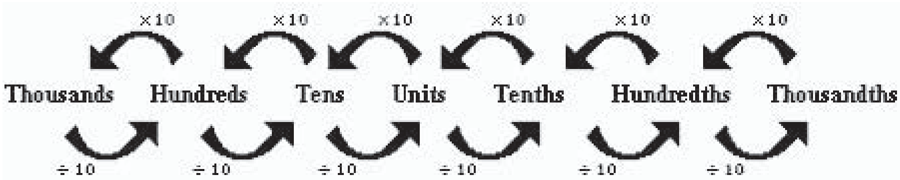



Figure 1. Multiplicative relationships in the base 10 number system (Steinle, Stacey, & Chambers, 2002, CD ROM).

Young children have a great capacity for constructing meaning for numbers (Hughes, 1986; Burns, 1994; Kamii, 1985; Ross, 1989). Based on this observation, it could be assumed that children will acquire the mental images and connections that enable them to work with larger numbers and more complex mathematical operations in meaningful ways as they grow older and as they move through the school system, but this appears not to be the case (Thomas, 1996). Most students learn to follow various procedures which are based on the place value system, but they do not understand the system deeply enough to invent alternative methods when appropriate or to deal with larger numbers outside of their common experience (Thomas, 1996). Even when students can competently perform algorithms, their understanding of why they do it (the conceptual understanding) is often lacking. Many students do not make the connection between the partitioning of a number into tens and ones and the decomposition of numbers in written algorithms (Fuson, 1986, 1992; Fuson & Briars, 1990; Wearne & Hiebert, 1988). Irwin (1996) suggested that many older students and adults have a very weak grasp of decimal fractions because they have attempted to build their ideas on shaky foundations about whole numbers and about fractions. This view was supported by Baturo (1998) who researched the interaction between available and accessible place value and re-grouping decimal number knowledge of Year 5 students, and found evidence of students using over-practised rules, and impoverished pre-requisite whole number and fraction knowledge. Results from the Learning Decimals project (Stacey & Steinle, 1995–1999) indicate that students' misconceptions about decimals are a significant problem. Data collected in this project show that approximately 45% of Year 9 students, and 40% of Year 10 students, were unable to compare decimals correctly. In the long term there is a general trend towards expertise, but as many as 40% of students may retain their misconceptions about decimals. As noted by Steinle, Stacey and Chambers (2002),

> misconceptions arise naturally, from students being unable to assemble all the relevant ideas together or from limited teaching, but students can easily be helped to expertise. Even a small amount of targeted teaching makes a difference. (CD ROM)

Developing knowledge about the number system

Much has been written about the development of understanding as opposed to the mastery of facts and procedures. Many students appear to know more than they actually do because they are able to give the correct response or perform a calculation correctly by relying on rules they do not understand (Thomas, 1996; Steinle & Stacey, 1998). Baturo (1998) asserts that "correct individual performance is possible without understanding and that availability of knowledge does not mean that it will be used (accessed)" (p. 96). The assumption that students have developed relational understanding or that they have generalised their knowledge, when in actual fact their thinking is still very context specific, can lead to a number of problems. When, for example, a teacher observes correct responses and assumes that the student has understood a concept, the teacher is likely to move them onto new learning experiences. However, if the mental images and relationships to which the student is trying to connect new knowledge are poorly developed, future constructs are weak and confused, and misconceptions may arise (Steinle & Stacey, 1998). Secondly, if students can execute a procedure but do not have understanding, it is unlikely that they will be able to use this knowledge when it is called upon in a new situation (Willis, 2000; Baturo, 1998).

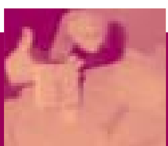
Thomas (1996) portrays the growth in constructing knowledge about the number system as “unpredictable, involving regressions as well as progressions and being, more often than not, non-linear” (p. 93). Whilst it remains important for teachers to know about broad stages of development, it is also important to stress that not all children develop in the same way or at the same time, nor can we assume that “the same early indicators and sequencing are equally appropriate for all children” (Willis, 2000, p. 32). When this is not recognised, it is easy to label these children as “behind” or as being “at risk (of not meeting the national numeracy goal). It is also easy to move these students through learning experiences that, instead of building on their existing knowledge, may actually undermine their current knowledge, and hinder the development of further knowledge. These findings by Willis (2000, p. 32) indicate that “differences between children in their learning of mathematics can neither be explained nor accommodated by variations in the pace at which they develop certain mathematical concepts. Rather, there may be differences in the very nature and sequence of their development of mathematical ideas.”

A number of research studies have identified relevant knowledge that plays a crucial contributory role in the development of an understanding of place value. Student’s levels of counting are significantly related to their knowledge of, and ability to explain, place value (Boulton-Lewis, 1996; Rubin & Russell, 1992). Research by Baturo (1997) concluded that an understanding of the multiplicative structure of the base 10 number system is a determining factor in differentiating high performance. To understand decimal numbers, students need to reconstruct their ideas of whole numbers, and fractions, to include decimal fractions (Irwin, 1997).

Use of concrete representations

Many studies have investigated the use of concrete materials; for example, pop-sticks and Multi-Base Arithmetic Blocks (MAB), in developing understandings about place value with mixed findings. Studies by Thomas (1996) and Thomas and Mulligan (1999) revealed limited understanding by many students and raised queries as to the value and effect of the use of MAB. It would appear that “children have learned to interpret certain concrete materials (bags, blocks, bundles, etc.) as representing the number system but have not reached the general level of understanding needed to interpret unfamiliar groupings (e.g., circled marks) in the same way” (Thomas, 1996, p. 97). The use of place value blocks (e.g., MAB) does not necessarily result in the development of powerful models of the number system (Price, 1998). The structure of the number system is not inherent in the materials themselves, but in the relationships that exist between the blocks, and these need to be constructed in the minds of the learner (van de Walle, 1990).

Boulton-Lewis (1996) investigated the use of representations of number by everyday materials, structured


or semi-structured materials as concrete embodiments of numbers. Boulton-Lewis noted that such representations are useful because they "mirror the structure of the concept and the child should be able to use the structure of the representation to construct a mental model of the concept" (p. 76). Practice in exchanging unit materials for sets of ten is an important activity for children in understanding the base 10 system, but we must be aware of the limitations. If students are to benefit from using concrete materials, they must be sufficiently familiar with the materials that the mapping of representations into the base 10 system of numbers is automatic. Regular use and discussion of the models can assist students in this process of mapping the concrete materials and the number names into numeric symbols and conventional names (see, for example, Hart, 1989).

Baroody (1989, p. 5) suggests that the use of materials alone does not result in better outcomes for students. Rather, learning will only be enhanced when the learning "experience is meaningful to the student and they are actively engaged in thinking about it". Price (1998) states that the use of place value teaching aids needs careful planning, and students' developing understandings need to be carefully monitored. We cannot assume that students are making sense of number representations in the same way that teachers do. Thompson (1994) suggested that research studies that give attention to the broad picture of the teaching/learning environment are needed to discern reasons for the success or failure of instructional use of concrete materials. The contradictions (among findings of different studies) may be due to aspects of instruction and students' engagement to which the studies did not attend. Evidently, just using concrete materials is not enough to guarantee success. We must look at the "total instructional environment to understand the effective use of concrete materials – especially teachers' images of what they intend to teach and students' images of the activities in which they are asked to engage" (Thompson, 1994, p. 556).

Aims of the research

This study attempted to build upon the work of previous studies by exploring ways that teachers could give students "more active help in developing an adequate understanding of the structure of the number system" (Thomas, 1996, p. 103) in a whole class setting, and to "find effective transitional experiences to bridge the gap between the use of concrete materials and formal written algorithmic procedures" (Hart, 1989).

The aim throughout the project was to help *all* students in the class *understand* the number system. The project operated from the belief that a relational understanding (Van de Walle, 1990) of the number system will lead to the development of higher levels of numeracy. Relational understanding is the process of connecting mathematical concepts and relationships (conceptual knowledge) with the symbols, rules and procedures that are used to represent and work with mathematics (procedural knowledge). Relational understanding is evident when the rules or procedures of mathematics have a conceptual and meaningful basis and the concepts can be represented by appropriate symbolism. Understanding is "what students need for next week and the week after, next year and the year after. It is sustainable learning. It is the only kind worth spending their time on – anything less than understanding isn't worth the risk" (Willis, 2000, p. 33).

There are many activities recorded that rely on the use of concrete materials in order to help students develop ideas about the number system. In this research project, the Base Ten Game (Pengelly, 1991) was investigated. The activity was selected because it models the structure of the number system and it uses concrete materials.

An intended outcome of the project was to provide a set of guidelines for teachers on how to use the Base Ten Game most effectively. This involved identifying and clarifying the following components:

- The modifications project teachers made to their use of the game to ensure that relational understanding of the number system was achieved;
- The other learning activities used by project teachers to complement the game.

The Base Ten Game

The game involves students using a place value board and concrete materials to develop an understanding of the structure of the number system and to learn to operate on numbers using this structure. To play the most basic version of the game, the student rolls two dice, adds the numbers shown, and collects that quantity of pop-sticks to add to their game-board, which is ruled up into place value columns (see Figure 1). The only rule of the game is that there can be no more than nine items in any one column. Once there are more than nine sticks in the units column, ten sticks are combined to make a bundle of ten sticks which is then placed in the tens column. The remaining sticks are left in the units column. When there are more than nine bundles of ten in the tens column, these are combined to make a bundle of ten tens. Rubber bands can be used to hold the bundles of sticks together.

Teachers have embraced this game because it is versatile enough to be used to develop relational understanding of the number system for all learners, from those just beginning to count and write numerals, to those exploring very large numbers and decimal numbers. Through participation in the Base Ten Game, students have the opportunity to develop their ideas about place value, and to learn to operate on numbers using this structure:

> Instead of using materials to demonstrate the facts, skills and procedures of arithmetic, the (game) frames the learning by giving children experiences with the structure as a whole … By giving children access to models of the place value system they gradually build an understanding of it. (Pengelly, 1991, p. 7).

The Base Ten Game has the potential to maintain a student's interest over many years because it can be modified continuously to develop thinking in more complex ways. It facilitates the progression from concrete to semi-abstract to abstract thought. In the Junior Primary years, pop-sticks and rubber bands (or any materials that can be grouped into successively bigger groups of ten), are the main materials used. Figure 2 shows a Reception

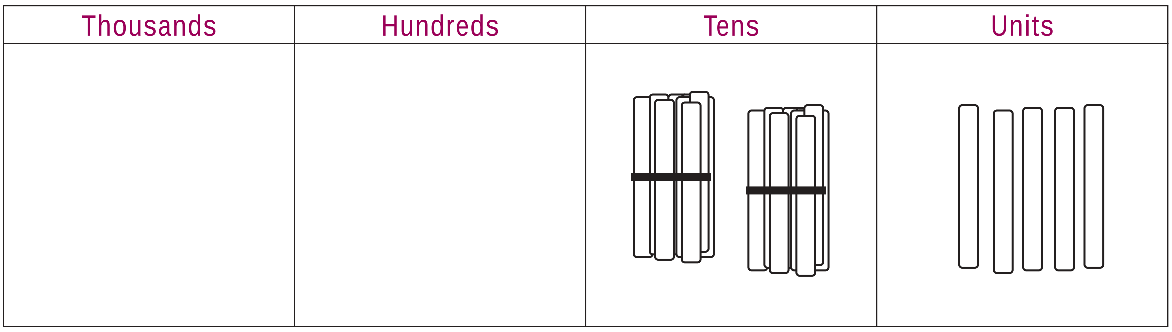


Figure 2. A game-board showing 25 pop-sticks.

class student counting pop-sticks for her units and tens game-board. Through participation in the game, students deal mainly with counting, place value, and addition and subtraction, and they may also use a version of the game to explore decimal currency.

Figure 3. A Reception student learning to play the Base Ten Game.

In Middle Primary years, the students continue to build on these ideas, and their experiences are broadened to include decimals, multiplication and division. At some stage during these years Multi-base Arithmetic Blocks (these are also called Dienes blocks or base 10 blocks) are introduced as an option for students to use instead of pop-sticks and rubber bands. Figure 4 shows a Year 3 student using pop-sticks to explore subtraction through the Base Ten Game.

The Base Ten Game continues into the upper primary years as students explore progressively bigger and progressively smaller numbers, and use the game to explore the metric measurement system. In Figure 4, a Year 5 student is using four dice and pop-sticks to explore continuous addition of larger numbers.

Figure 4. A Year 3 student using pop-sticks to explore subtraction through the Base Ten Game.

Figure 5. A Year 5 student using four dice to explore continuous addition of larger numbers through the Base Ten Game.

Throughout every year level, students have the opportunity to develop their own versions of the Base Ten Game to explore ideas and relationships of interest to them and to provide extra challenges through varying the use of the dice, (for example, by multiplying the two numbers instead of adding them), or the value assigned to the materials, (for example, by giving the largest bundle or block the value of one instead of the smallest).

Research Methodology

Participants

Nine primary teachers from five Independent schools in South Australia participated in this project. These schools represented the diverse range of communities within the independent sector. They included an isolated rural school with a high number of Indigenous students; a small and a large urban school; and two schools with high numbers of students from a low socio-economic background. The schools were supportive of their teachers being involved in the project.

Teachers were selected to be involved in this project on the basis of their participation in MLATS (Mathematics Learning and Teaching for Success)—a professional development course extending over three school terms. These teachers represented the year levels from Reception to Year 7, and were prepared to be involved in researching aspects of their practice that related to the development of knowledge of the base ten number system. All of the teachers were familiar with the Base Ten Game and had used it for a number of years in their classrooms.

Model

An action research model was adopted, where each participating teacher set their own research question, under the broader focus of exploring the use of the Base Ten Game to develop understandings about the number system. The model was developed in conjunction with Colin MacMullin, Associate Professor of Education at Flinders University, who describes action research as

> a systematic process whereby practitioners voluntarily engage in a spiral of reflection, documentation and action in order to understand more fully the nature and/or consequences of aspects of their practice, with a view to shaping further action or changing their situation preferably in collaboration with colleagues. (MacMullin, 2001)

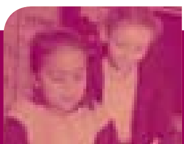
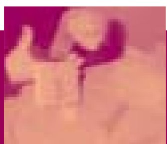
The research methodology enabled the project to focus on changes in student learning outcomes in response to changes made by the teacher in the classroom while respecting the diverse philosophies of the participating schools and teachers.

Individual teachers involved in the project framed and refined their own research questions (see Appendix 1) in relation to the broader question of how to maximise student understanding of the base 10 number system in a whole class setting. Investigations prompted by the research questions included ways of catering for the diverse learning needs of a wide range of learners, using peer conferencing to aid understanding, and linking the game to the four operations, decimals, fractions, and measurement.

Process

Project teachers met in Term 1 for a full day workshop where Professor Colin MacMullin introduced them to action research methodology. The role of reflective journals was presented as the major means of collecting qualitative data. Participants were introduced to the project's aims and overall research question, and began to consider a range of questions on which they might want to focus in their research.

Early in Term 2, Professor MacMullin and the project officer visited individual project teachers in their schools. The purposes of these meetings were to assist teachers in clarifying their research questions, to discuss the procedure for the action research component of the project, and to check the pre-test and confirm that it would be given to students within the following two weeks. (See Appendix 1: Teacher Research Questions).


Soon after these meetings it became apparent from discussions with the project teachers that they were struggling to identify the knowledge that they wanted their students to learn about the number system, and how the Base Ten Game could actually be used to develop this knowledge. This was surprising because, prior to the commencement of the project, the teachers had felt confident with their use of the game in helping their students learn about the structure of the number system. As a result of this feedback, a whole day workshop was held so that the teachers could discuss their own understandings of the number system, identify the procedural and conceptual knowledge that students need to develop in order to understand the structure of the number system and how it operates, and discuss how the game could be used to develop this knowledge across all year levels R – 7. In hindsight, this process should have occurred at the beginning of the project, before the research methodology was introduced, before teachers set their own pre-tests, and before they identified their research questions.

Throughout the project, teacher-researchers and the project officer met on average twice a term to clarify issues as they arose, share concerns, reflect on journal entries, and discuss their observations and experiences using the research questions as a focus.

Data

Quantitative data

Project teachers developed or selected their own pre- and post-tests to collect quantitative data. These tests were developed with reference to a wide range of testing procedures, including the Basic Skills Test and the Western Australian Literacy and Numeracy Assessment. One teacher used the Decimal Comparison Test (Moloney & Stacey, 1996) to collect pre- and post-test data on Year 6 students' ability to compare decimals (see Appendix 2: Decimal Comparison Test). The data from this test enabled comparisons to be made with students of the same year level in other studies. This data is presented in Chapter 2.

Qualitative data

Qualitative data were provided by the teachers' research journals, a range of on-going classroom assessment activities (e.g. evaluation of student work samples, learning logs, student self-evaluations, focussed observations, student surveys, etc.), and an interview with selected students from each class (see Appendix 3: Post Research Student Interview). In addition, each teacher conducted a case study of several students throughout the project. These students represented a range of thinking levels, and also a range of target groups, and represented a variety of gender, ethnicity, socio-economic background, and language backgrounds other than English. Summaries of two case studies have been included in Chapter 2. Six months after the conclusion of the action research component of the project, the project teachers individually completed a questionnaire (see Appendix 4: Post Research Teacher Questionnaire) to reflect on the impact of the project on their own learning and on their students' learning.

All of the qualitative data were collated, sorted and coded, and analysed for emerging themes, and have been summarised in the next chapter.

Chapter 2: Results and Discussion

Quantitative data

Table 1 shows the grade levels of the project classes, the numbers of students in each class, and pre-test and post-test means for the test which was used by the teacher for that class. In all cases the children's performances on the pre-tests and post-tests exceeded their teachers' expectations. Substantial improvement over the period of the project occurred in all classes.

Although the tests were different for each class, the combined data provides a means of integrating the findings from the nine classes involved in the project. Teacher B, for example, used the task shown in Figure 6 with her Reception class.

Can you put out pop-sticks to match the numbers in each of the boxes?

Teacher	Grade level	Number of students	Pre-test mean (%)	Post-test mean (%)
A	R	20	29	88
B	R	10	37	69
C	1	30	48	67
D	2 / 3	17	17	89
E	4	30	74	96
F	6	29	57	76
G	6	11	62	83
H	6 / 7	22	47	63
I	6 / 7	19	61	76

Table 1. Grade levels, numbers of students, and pre-test and post-test means for the project classes

Name	5	8	4
10	1	7	3
9	6	2	

Figure 6. Teacher B: Sample item from the Reception class test.

One Year 6 class completed the Decimal Comparison Test (Moloney & Stacey, 1996). The Decimal Comparison Test asks students to circle the larger number from a series of pairs of decimal numbers, as shown, for example, in Figure 7.

Put a ring around the larger number in these pairs

4.8	4.63
0.4	0.36
0.100	0.35

Figure 7. Sample items from the Decimal Comparison Test (Moloney & Stacey, 1996).

Use of this test enabled a comparison of the performance of these Year 6 children with data obtained from other studies. The test was administered to the Year 6 children at the start of the research project (March, 2001), at the end of the research project (August, 2001), and one year later (November, 2002) after the students had completed Year 7.

Students who responded correctly were referred to as *Task Experts*. Incorrect responses were classified according to the student's underlying misconceptions and the associated strategies as described by Resnick et al. (1989):

Whole number strategy: The number with the most digits to the right of the decimal point is always the largest

Fraction strategy: The number with the least digits to the right of the decimal point is always the largest

Zero strategy: As for the whole number strategy, but with added knowledge that a zero in the tenths column makes a decimal part small.

Students not fitting into any of these three groups were categorised as *Unclassified*.

Table 2 shows the pre-test and post-test data for the Year 6 class, together with results from similar aged students in studies by Steinle and Stacey (1998) and Helme and Stacey (2000), classified according to *Task expert*, *Whole number strategy*, *Fraction strategy*, or *Unclassified*.

Table 2
Percentage of students using each strategy on the Decimal Comparison Test: Various samples

	Task Expert	Whole number strategy	Fraction strategy	Unclassified
Project Class				
Grade 6 (*N* = 29)				
Pre-test (March, 2001)	25	45	22	8
Post-test (August, 2001)	75	13	8	4
Grade 7 (*N* = 29)				
Delayed Post-test (November, 2002)	84	8	4	4
Comparison Data				
Steinle & Stacey (1998)[1] No intervention				
Grade 6 (*N* = 319)	52	17	12	19
Grade 7 (*N* = 814)	54	13	14	19
Helme & Stacey (2000) Short intervention				
Pre-test Grade 6 (*N* = 47)	57	6	6	30
Post-test Grade 6 (*N* = 47)	81	6	0	13

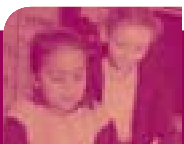
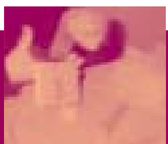
1: Unclassified calculated from other figures.

By the end of the project 75% of the students in this Year 6 class were using an expert strategy to compare decimals. This was a marked improvement from the 25% of students who were using an expert strategy at the beginning of the project. This was also a greater percentage than the 52% who were using an expert strategy in the study by Steinle and Stacey (1998). Additionally, the data from the project shows that the percentage of students using an expert strategy grew from 75% to 84% over the following year. This compares with 54% of Year 7 students who were found to be using an expert strategy in the Steinle and Stacey study, and is comparable to the improvement noted in the short intervention study by Helme and Stacey (2000).

These results indicate that students can make progress in their learning beyond what one would expect with normal growth and development, and furthermore, this learning can be retained over time. Analysis of the qualitative data gives an insight into how the Base Ten Game contributed to this progress.

Qualitative data

By focusing on features of the number system during the professional development activity, the teachers developed their own relational understanding that enabled them to identify what they wanted their students to know. They


explicitly identified ways they could use the game and other complementary activities to help students develop their knowledge. Analysis of the qualitative data collected throughout the project revealed a number of specific factors that related directly to the development of relational understanding about the number system:

1. The teacher's knowledge of
   - the patterns and relationships of the number system and how these impact on a whole range of mathematical ideas;
   - what constitutes relational understanding of the number system;
   - how the game might be used to develop relational understanding of the number system.
2. The extent to which the teacher
   - accurately established (through a variety of strategies) what a student already knew about aspects of the number system;
   - interpreted the data gathered about students' current constructs in terms of what they already knew and what they needed to learn;
   - used this data to plan learning experiences for each student to build on their current knowledge.

The three dot points in 2 (above) repeat continuously and constitute the teaching/learning cycle.

Analysis of the qualitative data also revealed other aspects of the total instructional environment that contributed to students' learning. These features are more general aspects of the overall management of the learning environment and are the foundation on which the teaching/learning cycle operates. These features are discussed in Chapter 3.

Individual case studies

The two children, Nathan and Jessie, described in the following case studies were members of the same Year 6 class. Their names have been changed to protect their identity.

Nathan

At the beginning of this project, Nathan was an eleven-year-old boy who believed that mathematics was too difficult for him and that he was unable to successfully participate in it. He would frequently make comments such as "I can't do maths. I don't understand anything in maths. I'm just not good at maths and I never have been." His parents would make the same comments about him and appear to accept that his poor results were unavoidable and expected. He had developed many avoidance and off-task behaviours during mathematics lessons—he took a long time to find his books and equipment and often worked on the floor where he was out of the teacher's line of vision.

Nathan believed mathematics was a subject where success depended upon the memorisation and following of rules and procedures. He would often say, "Tell me the steps again … I've forgotten." It was as if these steps were the magic formula – provided he could remember them. His problem was that he could neither remember rules; nor had he developed the conceptual understanding as to why these rules worked. He was still very context-specific in his thinking.

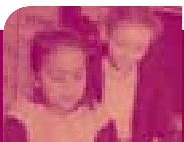
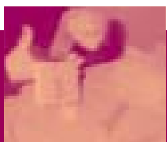
Nathan did not have a clear sense of the structure of the number system. He could not explain the relative size of numbers For example, he was unable to say how much bigger one thousand was than one hundred. He lacked place value language beyond the thousands column. He lacked understanding of the multiplicative features of the base 10 system. For example, he didn't realise that numbers could be grouped as quantities of ten and ones, for example, that 10 tens made a hundred, that 10 hundreds made a thousand, that 100 tens made a thousand. He didn't know that numbers have many equivalent representations. For example, that 32 could be made up of 3 tens and 2 units, or 2 tens and 12 units, or 1 ten and 22 units. He could not consistently or accurately order numbers to 1000. When, for example, he was asked to sort numbers according to those that were close to 1000, 1500 or 2000, his responses were largely based on guessing. Most of the time he would just say, "I don't understand" and then begin to mumble so that you couldn't clearly hear any of his responses. His counting skills were also extremely poor. This was evident when he was observed playing the Base Ten Game. Whenever he had to work out the total on his game-board he would always count in ones. He also tended to skip numbers when getting close to one hundred, for example, 87, 88, 92. His ability to visually and orally read numbers extended to 9999 but he struggled beyond this point. When asked to read larger numbers (e.g., 10 456) he would often begin at one thousand, or if he thought the number was very large, he would start from one million. Thus in the above case, he might have said 1 thousand 4 hundred and fifty six, or, alternatively, 1 million, 4 thousand and then he would mumble the rest of the number so that he couldn't be heard.

Nathan didn't have an understanding of the relative size and quantity of the MAB equipment being used. Thus, he might look at a "thousand" block and call it one thousand, but when asked how many units in this block he would say 700. He would point to each face and count by one hundred and then add an extra hundred for good measure.

Nathan needed time to develop counting skills. He needed activities that would help him to count backwards and forwards and to skip count. He needed to explore the way the number system was structured and how the materials he was using modeled the structure of the number system. This included exploring how the MAB blocks were structured.

The avoidance tactics that Nathan had so carefully developed over the years were blocking learning opportunities for him, and therefore had to be dealt with before concentrating on some of the other learning issues. The first term was devoted to developing strategies of accountability to reduce avoidance tactics. The first step was to give Nathan an equipment checklist to follow each morning. Spare pencils and rulers were always available on the teacher's desk. At the same time, Nathan was given very specific and small tasks to complete before he took the next break. For example, the teacher might have said, "Today when you play the Base Ten Game, you must show at least 10 dice throws". Alternatively, he might be given a target such as: "Today you need to play the Base Ten Game until you get a total of 200 on your game-board." The teacher developed task cards that he could paste into his book, so that tasks were given in both an auditory and visual format. Work that was spread over a number of pages had to be rewritten and a bright post-it note in his book each day reminded him to continue on the same page.

During term two, the teacher reported how she began using the Base Ten Game more efficiently. The first adaptation she made was to ask Nathan to use pop-sticks, rather than MAB equipment, to play the game. Previously this had not been a choice, for from the beginning of Year 6, all students were required to use the MAB. Re-introducing pop-sticks, after students had already began using MAB, allowed a "value" to be added to the equipment. Some students who needed


to use paddle pop-sticks refused because they saw it as an indication of their lack of success. This created a dilemma for the teacher:

> 'As a classroom teacher I knew there were some students who would benefit from using pop-sticks but, without destroying their self-esteem, it was difficult to impose this equipment onto them. In hindsight, I should have presented both paddle pop-sticks and MAB equipment as a choice from the beginning. In doing this, students would have seen this equipment as just another choice—equally valid and equally valued.'

Fortunately, Nathan was quite content to use the pop-sticks. In fact, he really enjoyed bundling them into groups and tying them with rubber bands. His task was to roll one dice, take the corresponding number of pop-sticks and place them onto his game-board. The only rule was that he was unable to have more than nine in any one column. Each day he needed to continue from his last total. At least three times a week the teacher would sit down next to him and watch, listen and question. Sometimes this was for ten minutes, sometimes it was for the entire lesson. Although the time that the teacher spent with Nathan was very intensive, it was also necessary, because it brought in a great deal of accountability not only to complete the task, but to be able to understand it. During the first week of bundling and counting, Nathan would begin counting his total from one each time. But he soon gained confidence and was able to continue counting from his previous total. The activity helped develop his counting skills, and especially his ability to count on in groups. This activity allowed him to succeed because the numbers he was using were always low—below 500. It also allowed him to explore simultaneously the conceptual and procedural understandings of our number system—it allowed him to think in groupings of ten, rather than just in ones. There was a deliberate use of language as part of the learning process. Thus, for example, he may have reached a total of 64 and would then have to verbally tell another person that this meant he had 6 groups of ten and 4 ones. Nathan was challenged and supported by this activity for four weeks.

At the end of term two, Nathan made a significant discovery. He ran up to his teacher and said, "I have just learnt that ten groups of ten make one hundred". From this discovery, Nathan's understanding progressed more quickly. He finally began to understand how our number system is put together and could make generalisations about the structure of numbers. Thus, if ten groups of ten made a hundred, he guessed that ten groups of one hundred made a thousand. His ability to count by bundles, to regroup, to recognise the place value concepts of the numerals and his ability to orally state a number improved dramatically. He was confident to write a number on his board, model it, and explain its place value groupings. In whole class sharing, he began to share his work and would volunteer to read larger numbers to the rest of the class. By the end of the project, he was confidently working with numbers from 0 to 100 000. His application of understandings to the four operations also improved dramatically. His confidence had increased, his on-task behaviour was wonderful and he was keen to participate and ask questions. Nathan's score on the post-test was 76%, compared with 23% on the pre-test.

The case of Nathan is significant for numerous reasons. He was a Year 6 boy who was struggling in mathematics. He had accepted that he was a "struggler" and so had his parents. His previous teachers also recognised that he was a "struggler", but they were unable to pinpoint his difficulties. Assumptions had been made that he could count, because surely that's what one learns in Junior Primary school. It had also been assumed that he understood that ten bundles of ten bundles of one made one hundred because he had been physically bundling ten bundles of ten to make one bundle of one hundred over the years that he had been playing the game. This raises important issues about the use of concrete materials—in the case of this research, students may play the game, bundling ten tens to make one hundred, but may fail to make the connection between their involvement in the game and the structure of the number system. Once Nathan was able to articulate that ten bundles of ten were equivalent to a bundle of one hundred, it was as if a shutter had been raised. He was subsequently able to progress to more complex levels of thinking about whole numbers and later, about decimal numbers, than had previously had been possible.

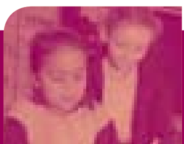
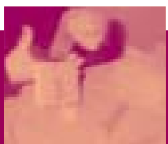
Assumptions were also made that Nathan was "ready" to use pre-grouped materials (e.g. MAB) as opposed to using un-grouped materials (e.g. pop-sticks). However, the act of trading ten tens for one one-hundred is conceptually much more complex than bundling ten tens to make one bundle of one hundred. It would appear that many students need to use pop-sticks for a much longer period of time than expected, before they are given the option of using MAB as an alternative to investigating place value and the four operations. Incorrect assumptions about students' understanding may result in some students being presented with concrete materials or learning activities that are conceptually too complex for them. The result of this being that students become over dependent on inefficient strategies such as memorising routines or procedures that hold no meaning for them. It is unlikely that these students cannot draw on this knowledge in situations that demand its use.

Jessie

Jessie was a student whose understanding of decimal numbers was particularly low. This was immediately apparent in the first activity in the decimals unit, where students were asked to circle the larger decimal number in each pair (Decimal Comparison Test, Moloney & Stacey, 1996). This gave the teacher a quick indication of the strategies students were using. Analysis of Jessie's answers revealed that she was using a fraction strategy—comparing decimals by interpreting the number with the least digits after the decimal point as being the largest number.

However, when Jessie was asked to represent a decimal number with MAB equipment, and to explain the strategy she used to tell which number was larger in value, she appeared to adopt an expert strategy—comparing numerals from the left and proceeding until one column was smaller than the next. The teacher reflected:

> 'I realised later that I had in fact led her with my questions to give me answers I wanted, and that in fact her understanding was still quite confused and weak. In discussing her strategy with her I would say "but look there are more tenths than hundredths in this number and so it has to be bigger, doesn't it?" There was no other response that Jessie could give except to say "yes". I would then move on, satisfied that she had a great strategy for ordering decimals—until the next lesson and Jessie would revert back to her original strategy. However, being in a hurry to get through the unit, I moved her onto playing the Base Ten Game with a decimal dice. On a cursory glance, I would have been impressed with her work. She was adding


and subtracting decimals efficiently. She was applying the same rules for addition and subtraction as she would for whole numbers. She therefore successfully met one of my learning objectives – to add and subtract decimals accurately. It wasn't until I came around and asked her to read her number that I realised she had no concept of a decimal. Her recorded total was 35.67. She first read it as 35 and 67 and then as 3 567. I pointed to the decimal point and asked her what it was. She said it was like a comma; it separated numbers. I asked her whether there were any numbers that were less than a whole in her total. She said "no". When I pointed to the numerals 6 and 7 and asked whether these were whole numbers, she said "yes".

'I was stunned—I had assumed that because she could add and subtract and play the Base Ten Game with a decimal dice that she understood the concept of a decimal. I also realised that she had no idea that decimals represented fractional parts. When I asked her to read her place value columns she would say tenths but when I asked her what a tenth was, she couldn't tell me. It became important to help Jessie develop the notion of fractional parts and their relationship to the place value system. Without this understanding, a concept of decimals is impossible. Many of my students did not understand what fractions were and thus couldn't relate decimals to fractional parts. In my yearly plan I had introduced decimals before doing a unit on fractions. I soon realised my mistake. We began a unit on fractions and only after that, in third term, did we pick up decimals again. This proved to be a much better sequence.'

The students were involved in many activities to develop a notion of fractions, for example, finding different ways to represent one tenth. They used measurement equipment such as capacity containers, lengths of rope, grids and MAB equipment, and fraction kits to show the same representation. In another activity, the teacher made a long number line with 0.5 written at one end and 0.8 at the other end. The students' task was to work in pairs and insert as many decimals, in order, as they could between the two numbers. Jessie added 0.6 and 0.7 and then said there were no others—she had begun to understand the concept of tenths but hadn't yet realised that decimals could represent even smaller amounts. She hadn't experienced decimals of different lengths. The breakthrough for Jessie came when the teacher challenged her to think about this, and her partner added 0.65. This was also an important step for the teacher in realising that students needed to order decimals in a variety of ways, to experience decimal numbers of varying length, and to have time to talk and question. By the end of the unit, Jessie's understandings had improved significantly. She could describe a decimal as being part of a whole and give a concrete example, for example, she was able to suggest that 0.2 would be like filling the capacity container to two tenths, and she could explain that the decimal point separated whole numbers from fractional parts. This progress was reflected in Jessie's responses to the items in the Decimal Comparison Test; her pre-test responses indicated the use of a fraction strategy compared with the task expert approach evident in her post-test responses.

Chapter 3: Discussion and Implications for the Classroom

Children's difficulties with the number system

Prior to being involved in this project , the project teachers had believed that participation in the Base Ten Game over successive years was a valuable experience for children, but they had assumed that the children would be learning about the number system just by playing the game. However, the research project highlighted many children who, despite being able to play the game, had not developed well-formed conceptual understanding of the number system, and in some cases, had developed misconceptions. This realisation came as a surprise to the project teachers and made them very aware of the need to accurately establish the constructs from which each child was operating.

As the project teachers engaged in the process of articulating the knowledge they wanted their students to acquire, the role of counting as fundamental to this knowledge became evident. An important part of the first two years of schooling is learning the counting sequence of number names and developing ideas about one-to-one correspondence and cardinality. Teachers in these year levels put a great deal of emphasis on these aspects of number. Teachers of higher year levels generally believed that by the time most students finish Year 1, they would be able to count. However, one of the most significant findings of the research project was the realisation by the project teachers that many students in their classes could not count by various amounts. For example, students in Years 2 to 7 showed a broad range of difficulties, including students who could not consistently accurately count up to ten pop-sticks; a student who frequently skipped the nineties, counting from 89 to 100 by ones, and from 80 to 100 when counting by tens; and students who used strategies for comparing numbers which did not work in all instances but who were unaware of their errors.

Identifying children's current number constructs

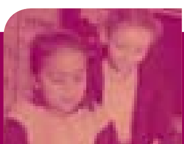
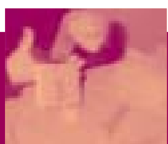
For learning in a whole class setting to be most advantageous to each individual within the class, the teacher must seek to establish the current constructs of each learner in order to provide a learning environment that will challenge and support them to develop more sophisticated knowledge. However, there are several features that influence the collection of reliable and accurate information of the students' current constructs:

Teachers' own knowledge of the number system

If the teacher's own knowledge of the number system is limited, it is very difficult to identify what others know and to teach them. One teacher, for example, expressed, "I didn't know the mathematics therefore I didn't know the questions to ask … I didn't understand or appreciate the way my students were thinking." The language and the applications that teachers use will influence students' conceptual development. If these are inappropriate or limited, teachers may actually cause confusion and the development of misconceptions in the minds of many students. Developing teachers' understanding of what it is that children need to know about the number system and the misconceptions students might hold is crucial in helping all students develop expertise.

The complexity of knowledge development

The development of knowledge is complex and does not always happen as expected; we make assumptions that students know certain things based on what we see and hear, or we assume that they can do one thing if they can do something else. One teacher reported, "By chance


I discovered Daniel could *count* by 10's to at least 400 (this was discovered through a class counting exercise), but he could only *write* numbers to 14." She assumed that because he couldn't write numbers beyond 14, that he couldn't count much beyond this either. If she had used this assumed knowledge and presented Daniel with activities to develop his knowledge of numbers to 20, she would not have been advancing his learning. In another example, Ned could read any number into the tens of millions. This could have prompted the teacher to make assumptions about his knowledge of large numbers. However, further investigations revealed that he could not count accurately by ones beyond 89, and by twos, beyond 20. If teachers proceed with presenting more and more challenging learning opportunities to students on the basis of invalid assumptions, the opportunity for some students to develop strong conceptual understandings may be lost.

Access to appropriate assessment tools

Appropriate assessment tools are not always available to teachers to help them make accurate judgements about a student's knowledge, in particular, their conceptual knowledge. Observations of a student at work can reveal important information about them as learners, however it cannot tell the observer what the student is actually thinking. It is easy to make judgements about a learner that are inaccurate. For example, when playing the game, a student might be seen counting out the correct number of pop-sticks, making bundles of ten, and placing the bundles and singles in their correct columns, but this does not tell us what the child is understanding about why they are making bundles, nor what they know about the total on the board. There are a number of tools available to give teachers information about the procedural knowledge of their students, but there are few available that help teachers make judgements about conceptual knowledge or relational understanding. The Decimal Comparison Test (Maloney & Stacey, 1996) is one example of an assessment task that enables teachers to quickly and accurately diagnose misconceptions about decimals, and more such tasks would benefit teachers in making these decisions.

Time constraints

Time constraints imposed by the school day mean that teachers have insufficient time to spend with each student, watching them, listening to them and getting to know their thoughts and ideas. Children also need time to explore the links and relationships between mathematical ideas, between mathematics and other areas of study and between mathematics and the world at home, work and play; to make connections between the many patterns involved in number; to share, evaluate and develop a range of strategies; to develop confidence and competence with the language of mathematics; and to reflect on their learning.

Linking concrete materials and conceptual understanding

Concrete materials are an essential feature of the Base Ten Game because they provide a visual model of the multiplicative structure of our number system. Initially, students spend much of their playing time concentrating on the mechanics of the game, for example, on counting pop-sticks to match the number rolled, on making bundles, and on knowing where to put the materials. This is an important process in beginning to learn about the game and students will require differing amounts of time to feel confident in knowing how the game is played. The same will occur when students use the game to explore new concepts, for example, when they first begin to explore numbers beyond one thousand or when they first begin to explore decimals.

Once they become more familiar and confident with the playing of the game, they can engage with the structure of the number system at increasingly more complex levels. Using concrete materials can help students learn about the structure of the number system, but their use does not appear to guarantee that relational understanding will occur. When students physically manipulate the materials they may develop a feel for the relative sizes of the various groupings of ten. However, this relationship is not always evident to the student. The teacher must provide contexts for consistent, deliberate and explicit exploration of the links between the conceptual and procedural knowledge as this appears to enhance students' understanding. In instances where attempts were not made to help students make these links, students were observed who had developed misconceptions about the number system, who were unable to see the links between the game and the number system, or who had developed superficial knowledge that could not be accessed when required and that was not sustainable over time.

Base Ten Game *Complementary activities*

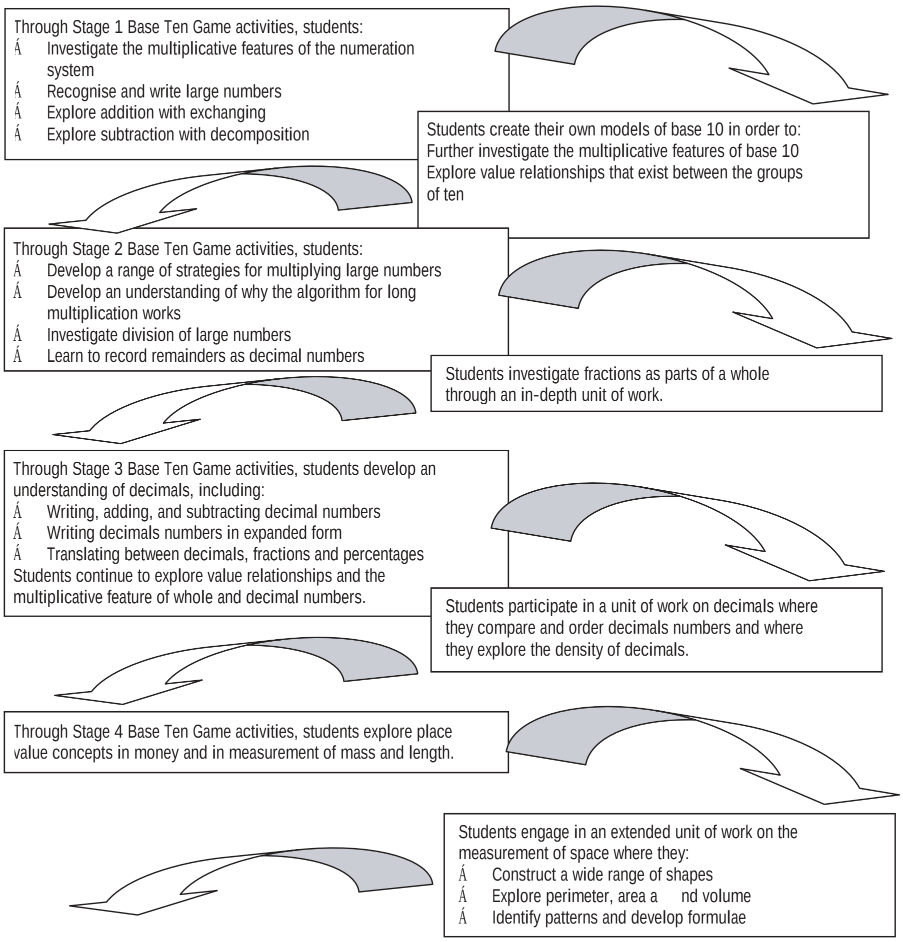
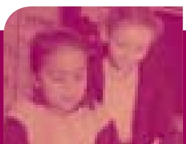
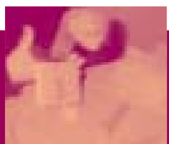


Figure 8. Base Ten Game and complementary activities in one Year 6 class.


Complementary activities

The Base Ten Game can play an important role in students' understanding of the number system, but it is also important that the game is complemented by other activities. Through this project a range of activities that served to complement the ideas and relationships being explored through the Base Ten Game were identified and trialled. Students will develop their knowledge about base ten in different ways. One activity may help one student, but not another, to develop new knowledge. Opportunities for students to make choices about which activities they will engage in, the nature of their engagement, the length of time they will engage in an activity, and how they will present their learning will enhance their understanding. Figure 8 attempts to illustrate the complex relationship between the Base Ten Game and other complementary activities in one of the project's Year 6 classrooms. The diagram represents the spiraling and interconnected nature of different versions of the Base Ten Game and a range of complementary activities. This relationship is not linear, nor lock-step. It is more like a patchwork of related activities. Students interact with different activities at different times and for different lengths of time. The curved arrows are used to represent this sense of moving in and out of activities as new concepts are explored.

Constructing Models of Base Ten

In a complementary activity for one of the Year 6 classes, students were required to construct their own models of base ten, based on volume, area and length (see Appendix 6 for details). This activity was posed after students had time to explore Base Ten Game: Stage 1 activities (see Figure 8, page 19).

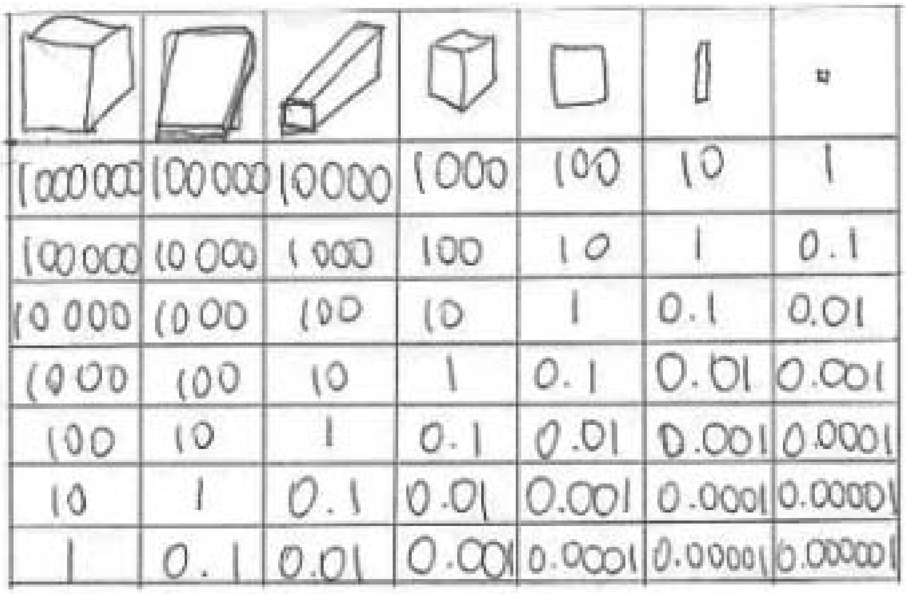
Initially, students were asked to make their own model of base ten by continuing the pattern of the four MAB blocks. Figure 9 shows the next two blocks in the series. These were created by a group of three Year 6 students.

Figure 9. Construction by three Year 6 students of a base ten model based on volume.

Figure 10. Recording of patterns by a Year 6 student.

After the students had made the next three blocks in the series they drew their blocks in sequence on paper, recording the dimensions of each block and describing any patterns they noticed. Figure 10 shows the recording of some of these patterns by one Year 6 student.

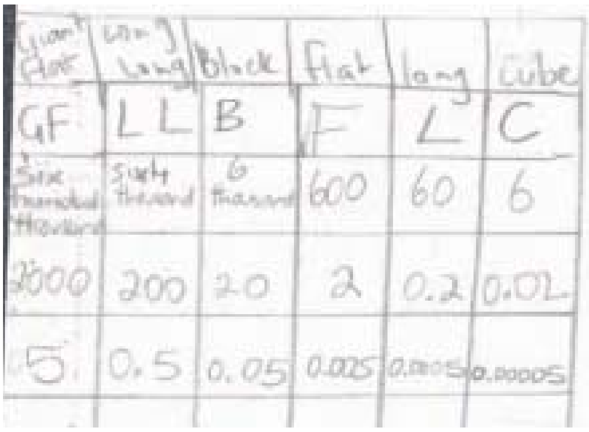
The next part of the activity asked the students to give each block in turn the value of one, and number the rest of the blocks in relation to the block with the value of 1. Figure 11 shows the recording of these relationships by one group of Year 6 and 7 students.



1000000	100000	10000	1000	100	10	1
100000	10000	1000	100	10	1	0.1
10000	1000	100	10	1	0.1	0.01
1000	100	10	1	0.1	0.01	0.001
100	10	1	0.1	0.01	0.001	0.0001
10	1	0.1	0.01	0.001	0.0001	0.00001
1	0.1	0.01	0.001	0.0001	0.00001	0.000001

Figure 11. Recording of the relative values of the blocks by one group of Year 6 and 7 students.

One student took this a step further and recorded the relative values of the blocks if they had values other than 1. Figure 12 shows the student's recording of these investigations. This student recorded the values of the other blocks when the smallest cube had the value of 6, when the flat had the value of 2 and when the giant flat had the value of 5. This recording has been copied onto the table presented in Figure 13.



Giant flat	Long Long	Block	flat	long	cube
GF	LL	B	F	L	C
Six hundred thousand	Sixty thousand	6 thousand	600	60	6
2000	200	20	2	0.2	0.02
5	0.5	0.05	0.005	0.0005	0.00005

Figure 12. A Year 6 student's recording of the relative values of the blocks.

Through subsequent activities, students were asked to make area and linear models of base ten, and to describe patterns and relationships noticed in these models. Students also used their series of blocks to investigate the value relationships between units of measurement (see Figures 17 and 18, Appendix 6).

Readiness for MAB

Prior to the research project, teachers assumed that most students were ready for the introduction of MAB around Years 3 or 4. However, this was found not to be the case. There were many students, even in the upper primary classes who were not conceptually ready for the MAB. These students still needed (and preferred) to physically construct, or decompose, bundles of 10. When teachers assumed that all students were ready for MAB and introduced the blocks to the whole class at the same time, many difficulties arose. In fact, it was noted that when MAB were introduced before a student was ready, future conceptual development appeared to be hindered. The findings from this project would suggest instead that MAB be offered as an alternative to pop-sticks (or any other ungrouped materials). This enables students to choose which materials to use and when to use them, and it allows them to explore the mathematical ideas in their own way at their own time without fear of being seen as "left behind" or "dumb".

Project teachers found it crucial to give general names to the MAB as soon as they were introduced, rather than using specific names with a numerical value. For example, the smallest block would be called a mini or a short instead of a one, the second smallest block would be called a long instead of a ten, and so on. This was very intentional so that when the MAB were being used, students could concentrate on the relationships between the sizes of the

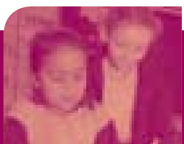
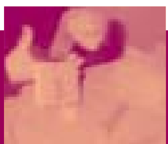
Giant Flat	Long Long	Block	Flat	Long	Cube
GF	LL	B	F	L	C
Six hundred thousand	Sixty thousand	6 thousand	600	60	6
2 000	200	20	2	0.2	0.02
5	0.5	0.05	0.005	0.0005	0.00005

Figure 13. A copy of the Year 6 student's recording presented in Figure 12.

blocks, and could change the values of each block in relation to the block with the value of 1. The fact that the blocks could take on different values was difficult for some initially, but it was also an important step in a student's conceptual understanding of the number system. They could see that the values of the blocks were not inherent to them, but in the relative sizes of the blocks. Additional learning activities, such as the one described above, were developed and presented alongside the game to help students develop confidence in describing the relationships between the blocks and their values.

The value of students' records of the game

Recording the game is difficult and complex for many students, and raised many questions about the purposes of recording, what should be recorded, when it should be recorded, who should record and so on. Teachers expressed mixed feelings about the need for students to record the game, and whether, for example, recording of each roll of the dice and the running total make any difference to the development of knowledge about the number system. The project did not reach any firm conclusions about the issue of recording, and this may provide the basis of further research.

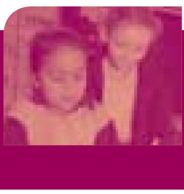

Chapter 4: Conclusion

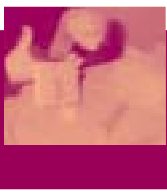
One of the major findings of the research was that teachers' own understanding of the number system was critical in enabling teachers to identify the features of the number system that they wanted their students to learn. Despite using concrete materials for many years prior to the research study, many children were observed to have difficulties with number concepts that teachers believed the children had been learning. By clarifying the desired learning outcomes, the project teachers were now better able to identify children who were having difficulty, and plan appropriate learning experiences for these children. Professional development which focuses on the children's conceptual understanding is likely also to assist teachers to develop their own understandings and then reflect on how this works in practice.

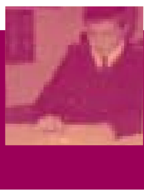
The project teachers recognised the need to accurately assess each child's number constructs and they used appropriate assessment tasks for this purpose. The Decimal Comparison Test (Moloney & Stacey, 1996) was particularly useful in the upper primary classrooms to identify children's misconceptions about decimals. Incorrect assumptions about children's understanding, based on inadequate assessment, can result in children being presented with inappropriate learning activities that are either conceptually too complex or too simple for them. The Base Ten Game was found to be a valuable core activity for students of all year levels who are still learning about the structure of the number system. Its usefulness was found to be further enhanced by the addition of complementary activities. These were used to support the ideas being developed through the Base Ten Game, and to investigate the same ideas in different ways. The project teachers developed ways to adapt their use of concrete materials to meet the individual learning needs of the diversity of students in their classes (see Appendix 5), as well as employing other learning activities to complement the learning that was occurring through participation in the game (see Appendix 6).

Use of a range of materials and activities in purposeful and explicit ways can contribute to the development of deep levels of understanding of the features of the number system. This project revealed the necessity of explicitly developing links between the concrete materials, the learning activities and the structure of the number system to support the development of relational understanding—many students do not automatically make these connections themselves. In instances where attempts were not made to help students make these links, some students developed misconceptions about the number system, some were unable to see the links between the game and the number system, or they developed superficial knowledge that was not sustainable over time.

Many students who initially were performing at a very low level gained confidence to participate in mathematics, and made substantial progress in their learning outcomes.







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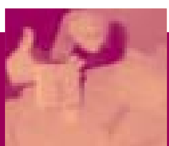


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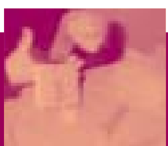
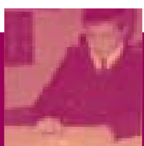
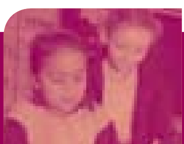
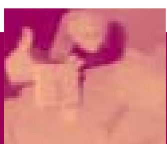



Appendix 1: Project teachers' research questions

Teacher-Researcher	2001 Year level	Research Questions
A	Reception	How can I cater for the range of individual children and termly intakes in my class when teaching the number system?
B	Reception	1. How can I help my students explore different possibilities in Base Ten activities? 2. How can I help my students discover the relationships between tens and ones?
C	Year 1	How can I use stories and the Bankers' Game to help my students make the links between learning in Base Ten and real life situations?
D	Year 2/3	How can I help students use effective peer conferencing to develop and construct their own understanding of base ten?
E	Year 4	1. How can I use the Bankers' Game to help my students develop a better understanding of the base ten number system? 2. How can I help my students develop a better awareness of their base ten knowledge and understanding?
F	Year 6	How can I adapt the Base Ten Game to cater for all my Year 6 students' needs utilising the four operators
G	Year 6	1. How can I use the Bankers' Game to help my students' progress in their Base 10 learning? 2. How can I use the Bankers' Game, and other activities, to help my students' progress in their Base 10 learning?
H	Year 6/7	1. How can I extend the scope and sequence of the Bankers' Game in a Year 6/7 context to: Explore the four operations +/- x/÷ place value decimals, fractions, measurement (relationships)? 2. How can I assist my students to develop a better understanding of place value, decimals and fractions in relation to Base 10?
I	Year 6/7	1. How can I better use the Banker's Game to help all my class progress with their Base 10 learning, especially in understanding place? Into decimals? 2. How better to use the Bankers' Game to help all students in my class progress in Base 10 learning, especially in understanding place value as it relates to whole number and decimals?


Appendix 2: The Decimal Comparison Test

Decimal Comparison Test

Moloney, J. & Stacey, K. (1996). Understanding Decimals. *The Australian Mathematics Teacher*, Vol. 52, no. 1, 4-8.

Complete the following task and glue this into your book.

Date:

On each line below there is a pair of decimal numbers. Put a ring around the larger one of the pair.

4.8	4.63
0.4	0.36
0.100	0.35
0.75	0.8
0.37	0.216
4.08	4.7
2.621	2.0687986
3.72	3.073
0.038	0.2
8.0525738	8.514
4.4502	4.45
0.457	0.4
17.353	17.35
8.24563	8.245
5.62	5.736

After you have completed this, try to explain what strategy you used to do this.

Appendix 3: Post research student interview questions

Give the student some equipment.

1. Can you show me how the Base Ten Game is played?

2. What do you like / don’t you like about playing this game?

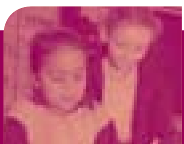
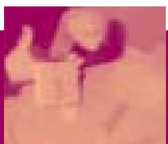
3. What did you learn from playing this game?

4. Why do we play this game / do this activity?

5. How is this game useful to you in other areas of mathematics?

6. *(Middle – Upper Primary)*
   Do you think that it is important to play this game / do this activity again this year? Why?

   (Junior Primary)
   Would you like to play this game again this year? Why?


Appendix 4: Post research project teacher questionnaire

∉ What were your overall feelings about this project?

∉ What were the highlights of the project for you?

∉ In what ways has your own mathematical knowledge developed as a result of participating in this project?

∉ What was the most relevant part of the research for you?

∉ In what ways were your current classroom practices affirmed?

∉ In what ways were your current classroom practices challenged?

∉ In what ways has this project impacted upon your school?

∉ What practices have been implemented in your school as a result of this research?

∉ In what ways have you applied the knowledge and skills you have learned?

∉ In what ways has this project impacted upon the learning outcomes of your students? What evidence have you for this?

∉ Other comments.

Appendix 5: Strategies for effective management of the Base Ten Game

Organisation of materials

- Ensure materials are easily accessible to students, for example, by having them organised in containers. This ensures that valuable playing (and learning) time is not lost with the distribution of materials.
- Instead of starting each day from 0, allowing students to continue on from their total of the previous day. Students keep their materials from day to day in a seal-top bag, which makes for easy access to the materials.

Assessing children's readiness

- Ensure that students have appropriate experiences to prepare them for playing the game For example,
  - before introducing the basic game, ensure students have had a range of experiences with "pre- game activities" (e. g., counting, bundling, matching, other trading games e.g. Lizard land, Geoff White, 2002);
  - before offering MAB as an alternative to pop-sticks and rubber bands, ensure that students have had appropriate experiences to explore the relationships that exist between the MAB, and had opportunity to explore and articulate value relationships;
  - before playing the game with decimals, ensure that students have a good understanding of whole numbers and fractions.

Creating links between concrete materials and the number system

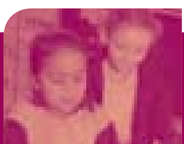
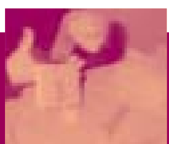
- Model the use of new materials, for example, the use of 0-9 digit cards (see Figure 13) , the names for MAB, the different possibilities for using dice, etc.;
- Conduct regular whole class sharing time to talk about what is working well for the students and what they are having difficulties with;
- Display examples of students recording, including the symbols and terminology that students are using as they learn.

During playing time

- Use observational checklists to focus on the development of specific aspects of the number system, for example, asking students
  - how they counted the total on the game-board
  - why numbers are written the way they are
  - what values they have assigned to each piece and why
  - how they decided to draw up their game-board and why
  - how they are using the dice and why

At the end of a designated playing time:

- Ask students to respond verbally in a whole class sharing time, or individually through writing, to a range of focus questions at the end of a playing time.
- For example, Figure 14 shows a Year 1 student's response to the following:
  - Write your last total as a number.
  - Write your total as a word.
  - How many of each group of ten were in your number?


- Draw a picture of your number.
- How did you count your total?
- How many different number sentences can you write about your total?
- Start at your total and count on.

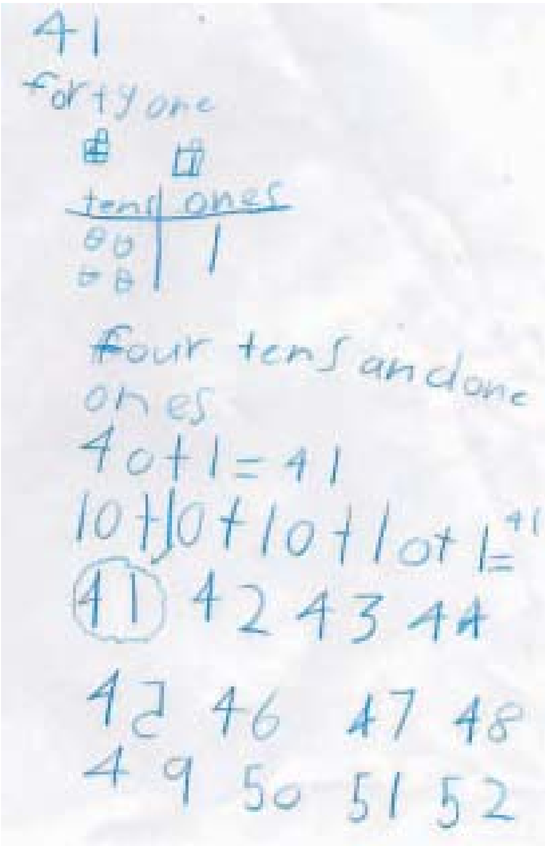


Figure 14. A Year 1 student's response to follow-up questions.

Figure 15 shows a Year 3 student's response to the following questions:

- Record your total
- Draw a picture of your total
- Record how many of each group of ten are in your total
- Write as many number sentences about your total as you can
- Start at your total and count on by ten.

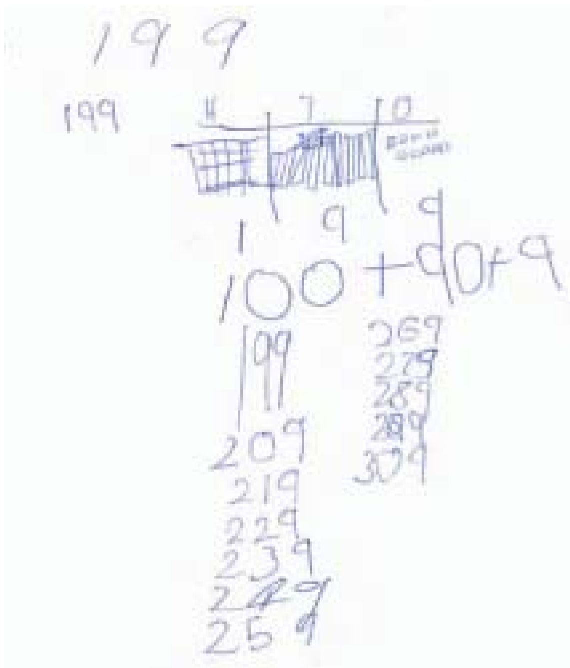


Figure 15. A Year 3 student's response to follow-up questions.

On another day this Year 3 student responded to the following:

'How many different ways could you represent your number using combinations of tens and ones? '

The student's response is shown in Figure 16.

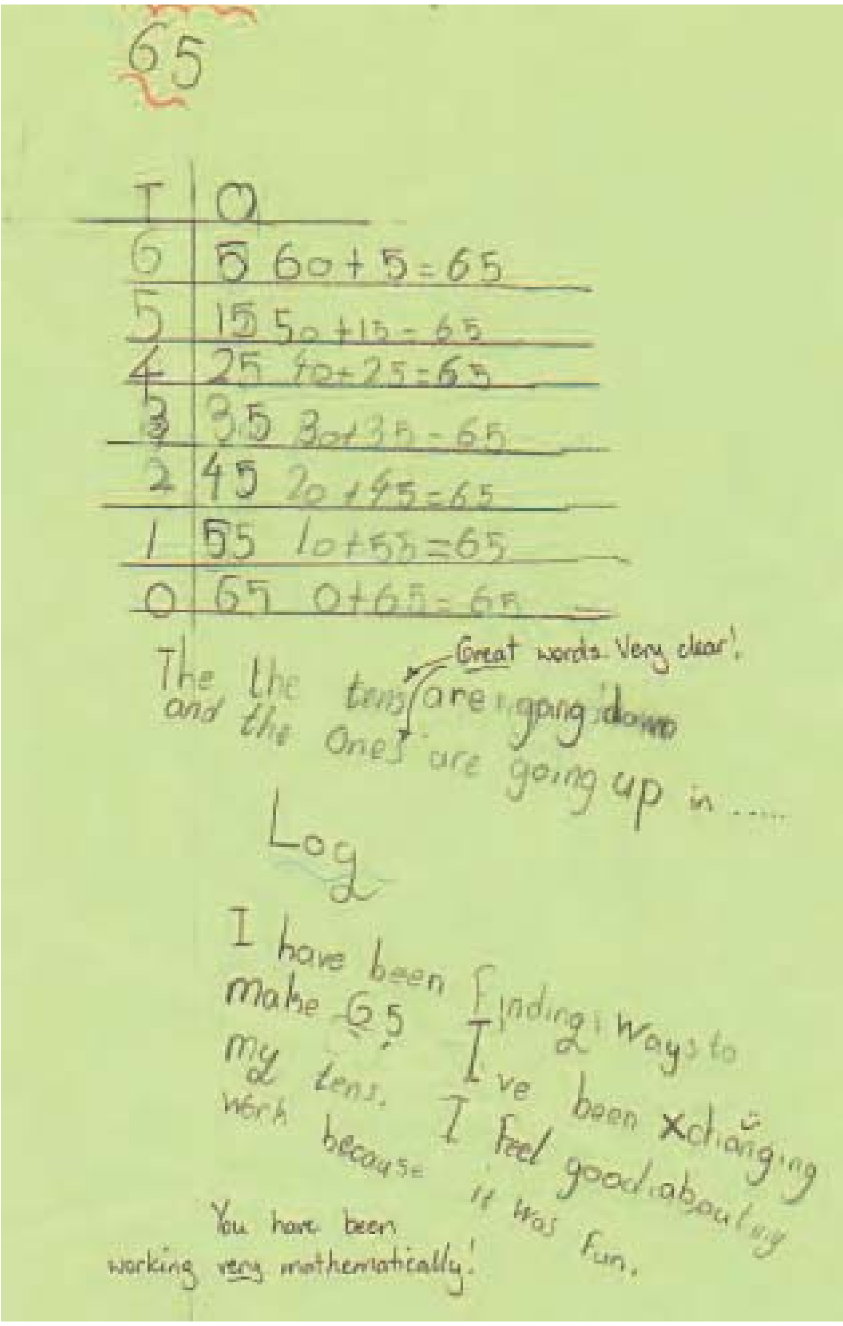


Figure 16. Another response by the same Year 3 student.

Figure 17 shows a Year 5 student's response to the following:

⊄# Write your number.

- Draw a picture of your total on your game-board.
- Write your number in words.
- Write your number in expanded notation.
- Start at your total and count on by a number between 0 and 1.

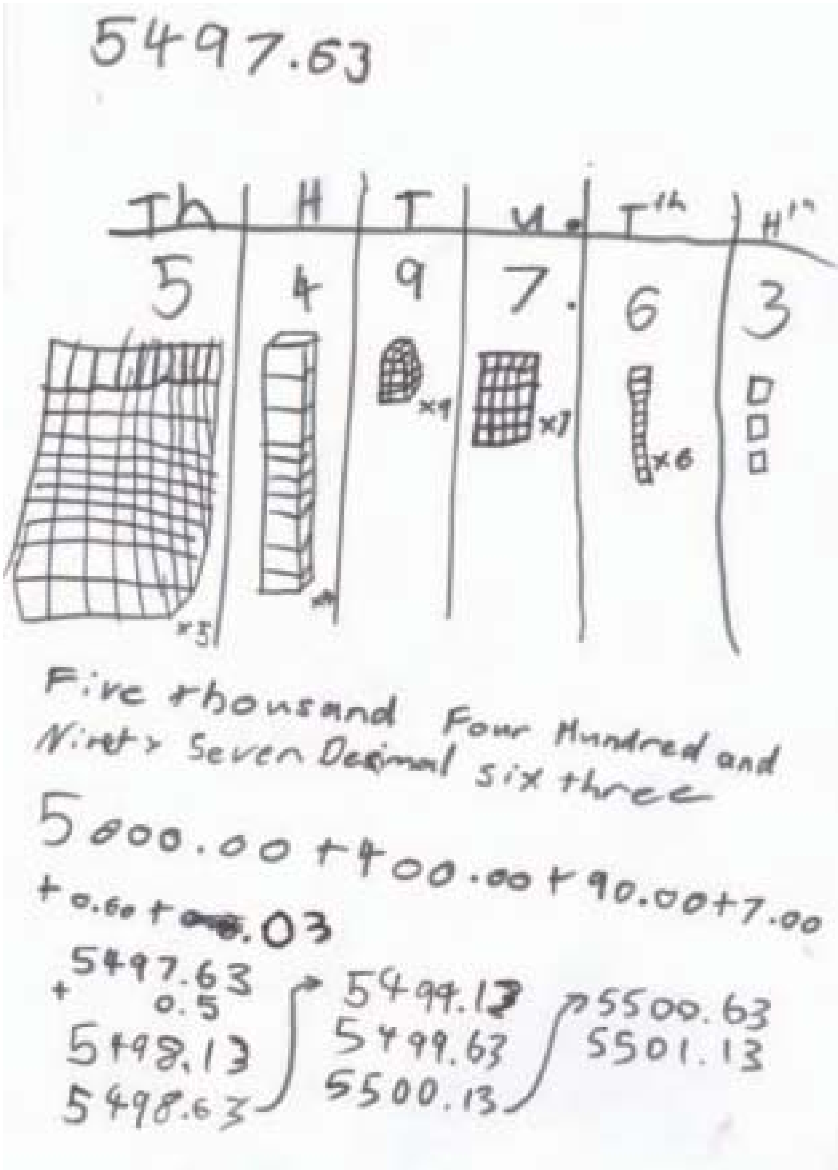


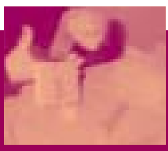
Figure 17. A Year 5 student's response to follow-up questions.

Other questions that have been posed to focus students attention on the total include:

- Why did you choose to count in this way?
- What number would come after it? Before it?
- What would the total be if you added one hundred more?
- What would the total be if you took ten away?
- Start at your number and count on by 5's.
- Tell me what number would be 100 more than this? 100 less than this?
- Take your total and count by 10 for the next 12 intervals. Now count by 100 for 12 intervals. Repeat but count by 1000's (always begin from your final total).
- Count backwards by 50 (or 10, or 0.5, or whatever is appropriate).
- Why is the number written in this way?
- Read your number to your partner.
- Read your partner's number
- Describe the value of each digit in your number
- ⊄# How far away is your number from 1 (or 100)?

Students may be asked to use their final total in a variety of ways at the end of a playing time, for example,

- see how many different numbers they can make using the digits in their total and rearranging them, and then placing these new numbers in order;
- ask students to read the same set of digits with and without a decimal point (324 and 3.24) and identifying the difference makes when decimals are present. Read each number to a partner. Draw a picture of each number. How much bigger is the largest number than the smallest one?
- ask all students to write their final total onto a pre-cut 10cm x 5 cm card and use this to place in order on a class number line;
- ask students to work in pairs and place their total on a number line and find as many decimals as they can between both points in 30 seconds;
- ask two students to place their totals on the opposite end of the board. Take it in turns to ask students to place a number between the two totals. Each time, the new number becomes the end point. For example, if the original number line was 2.4 ------------ 3.67, and someone puts up the number 2.5, the next student must put up a number between 2.4 and 2.5.


Using a range of materials to support learning

For example:

- give students 0-9 digit cards on rings (see Figure 18) if they are unsure about how numbers are written and what the digits represent.
- have number charts available if a student is having difficulty counting or writing numerals
- have a range of mathematics dictionaries readily accessible for students to use when they want information on the number system
- support counting by using the constant feature of calculators.

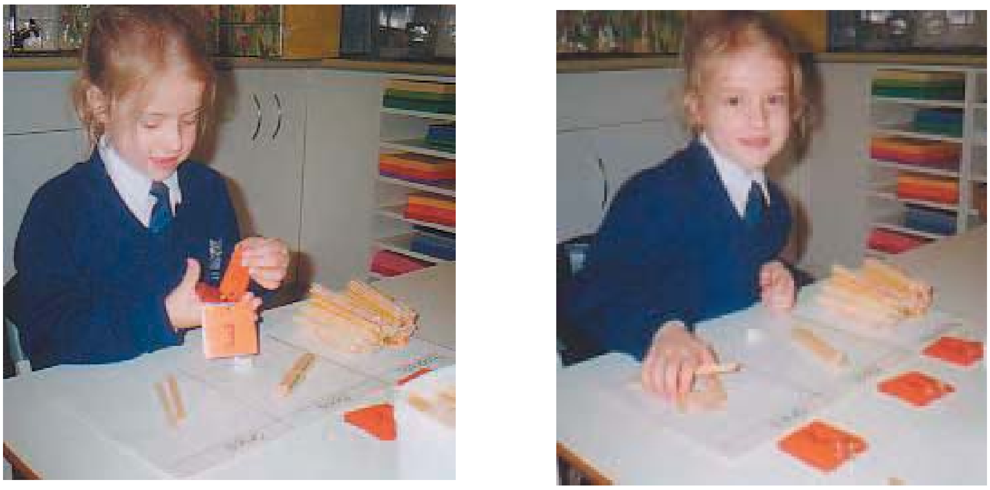


Figure 18. Using 0-9 digit cards on rings to support learning about how numbers are written.

Appendix 6: Samples of complementary learning activities used in the project

Constructing models of base ten

- Take the four different sized MAB and line them up. Predict what size and shape the next block in the series would be. Justify your decision.
- Use cardboard to make the next three blocks in the series (see Figure 9). Describe any patterns you find.
- Record your series of blocks on paper. Figures 19-21 show alternative ways that students recorded this information

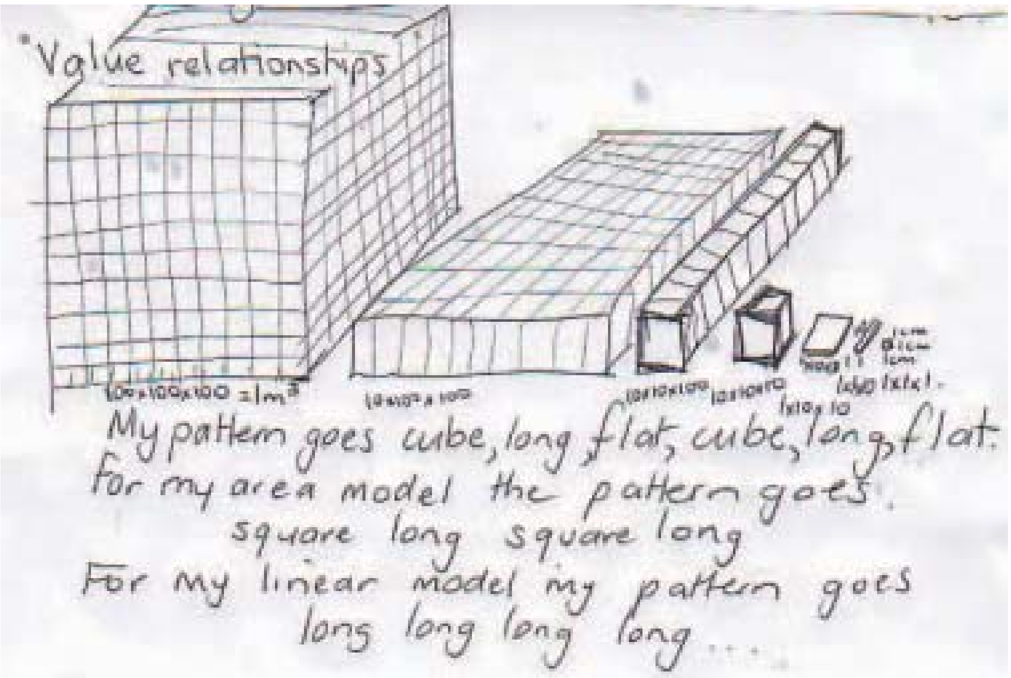


Figure 19. A student's recording of her series of blocks.

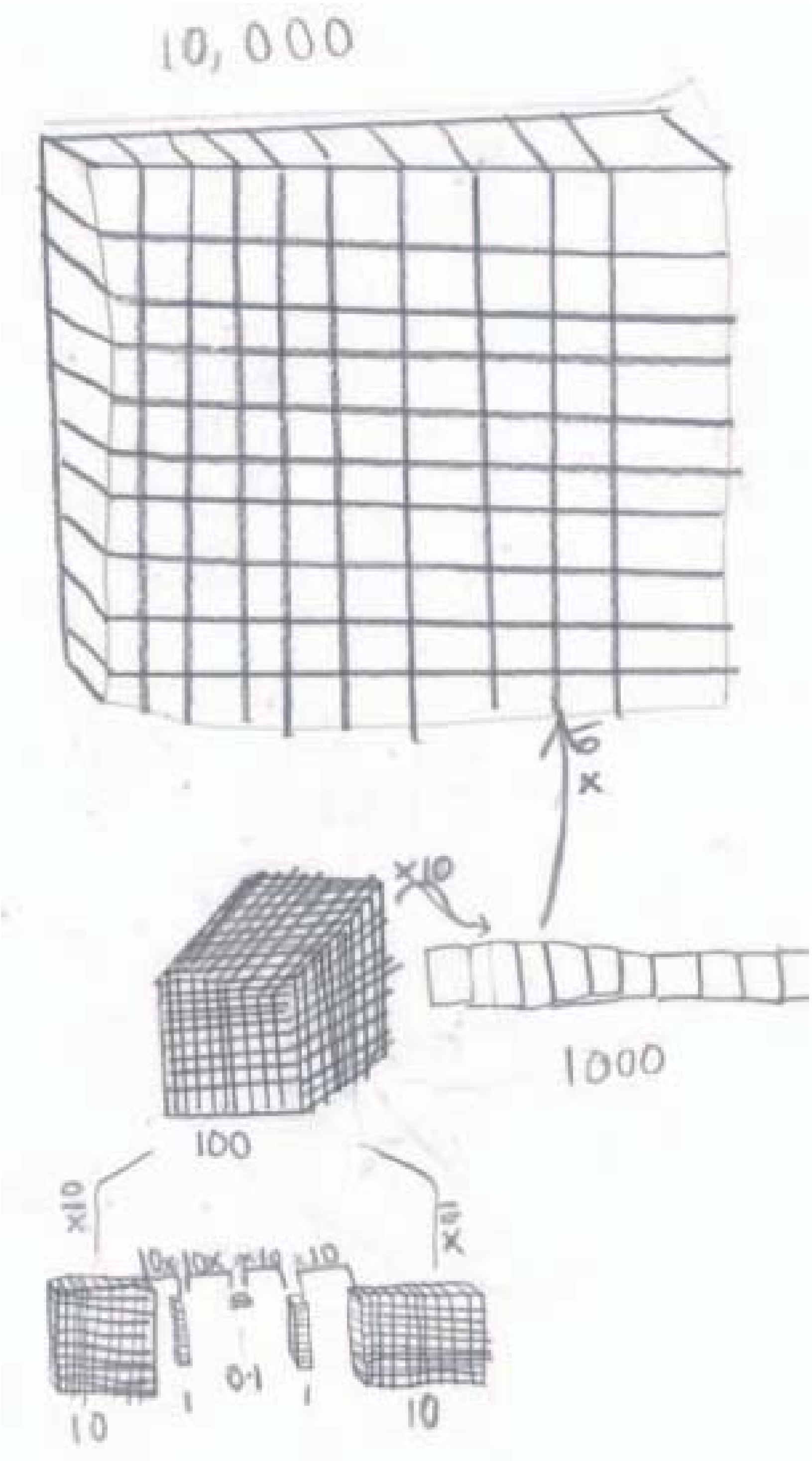
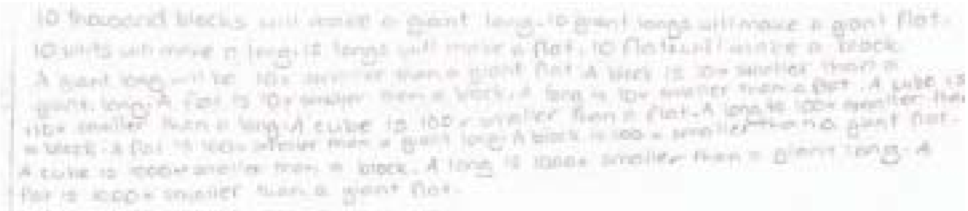


Figure 20. An alternative way of recording the relationships between the blocks.



‘10 thousand blocks will make a giant long. 10 giant longs will make a giant flat. 10 units will make a long. 10 longs will make a flat. 10 flats will make a block. A giant long will be 10 × smaller than a giant flat. A block is 10 × smaller than a giant long. A flat is 10 × smaller than a block. A long is 10 × smaller than a flat. A cube is 10 × smaller than a long. A cube is 100 × smaller than a flat. A long is 100 × smaller than a block. A flat is 100 × smaller than a giant long. A block is 100 × smaller than a giant flat. A cube is 1000 × smaller than a block. A long is 1000 × smaller than a giant long. A flat is 1000 × smaller than a giant flat.’

Figure 21. A Year 6 student explaining the relationships between the blocks.

Figure 22. Relating the volume model of base ten to the units used to measure length.

- Give each block in turn the value of one. What is the value of every other block in relation to the “one”? (see Figures 11 and 12).
- Use your series of blocks to investigate the value relationships between the units of measurement (see Figures 22 and 23).

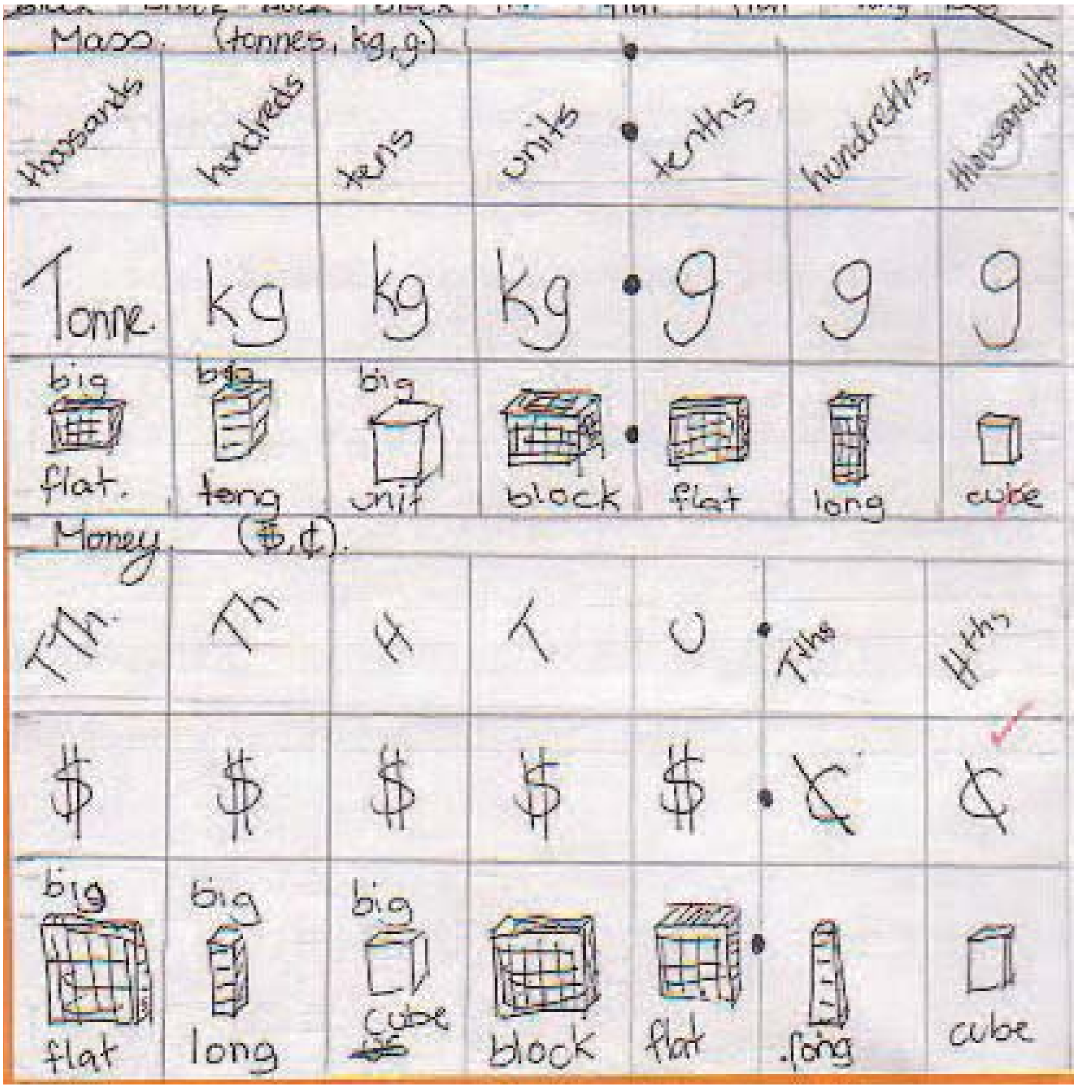


Figure 23. A student's work in relating volume models of base ten to mass and money.

- Make an area model of base ten. Make four pieces of your model. How is it the same and how is it different to the volume model?
- Make a linear model of base ten. Record and describe it.

A place value dice game

Ask the students to draw a box with a certain number of squares, for example three squares.

The aim of this game is to make the highest number by placing digits in the grid. A die is rolled twice as many times as the number of squares, for example, with a 3 square grid the die is rolled 6 times. The student may choose to use the number rolled, or choose to discard it. The students must make their decision about where to place the digit in the grid or to discard it before the next roll of the die. The game may be varied by the changing:

- the number of boxes (and hence place value positions)
- the numbers on the die
- the aim of the game (e.g., aim for the smallest number or the number closest to 10 etc.)
- from a whole number to a decimal number by adding a decimal point.

Numerous activities can develop from this game. For example, ask students:

- with the digits rolled, what was the highest possible number?
- with the digits rolled, what was the lowest possible number?
- with the digits that were rolled, what were all the possible numbers that could have been made? Order them from largest to smallest.